SPR 4106 Syntax and semantics in formal terms

Chapter 5 “Composing Extensions”: 5 Essentials

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Composing Extensions: Principle of Compositionality

> The extension of a mother is a function of the extensions of the two daughters and the way these two extensions are composed

- For any s, $[\![\mathrm{a} + \mathrm{b}]\!]_s$ is uniquely determined by
  - (i) $[\![\mathrm{a}]\!]_s$,
  - (ii) $[\![\mathrm{b}]\!]_s$, and
  - (iii) the composition principle in use between the two.

  Ideally, (iii) follows from the **types** of $[\![\mathrm{a}]\!]_s$ and $[\![\mathrm{b}]\!]_s$ (and the order of a and b is immaterial)

$$[\![\text{a} + \text{b}]\!]_s = \begin{cases} 1 \text{ iff } [\![\text{a}]\!]_s \in [\![\text{b}]\!]_s & \text{if a is an RT and b is a VP} \\ [\![\text{a}]\!]_s * [\![\text{b}]\!]_s & \text{if a is a TV and b is an RT} \\ [\![\text{b}]\!]_s * [\![\text{a}]\!]_s & \text{if a is an FN and b is an RT} \\ [\![\text{a}]\!]_s \cap [\![\text{b}]\!]_s & \text{if a is an A and b is an N}^{1} \end{cases}$$

VP = verb phrase, TV = transitive verb, RT = referential term,
FN = functional noun, A = adjective, N = noun

There are two versions of the principle **plugging in**, $*$:

- $R * y = \{ x : \langle x, y \rangle \in R \}$
- $y * R = \{ x : \langle y, x \rangle \in R \}$

Composing Extensions: Composition Principles

Alternative: reduce the two versions of $*$ to one

$$[\![\text{a} + \text{b}]\!]_s = [\![\text{b} + \text{a}]\!]_s = \ldots [\![\text{a}]\!]_s * [\![\text{b}]\!]_s = \{ x : \langle [\![\text{b}]\!]_s, x \rangle \in [\![\text{a}]\!]_s \}$$

if a denotes a relation between two individuals
(that is, it is a transitive verb or a relational noun or adjective)
and b denotes an individual (that is, it is a referential term)

Examples: *aimer*, *amante*, *amoureuse*

$$[\![\text{aimer} + \text{Chopin}]\!]_s = [\![\text{amante/amoureuse} + \text{(de) Chopin}]\!]_s =$$
$$\{ x : \langle [\![\text{Chopin}]\!]_s, x \rangle \in [\![\text{aimer/amante/amoureuse}]\!]_s \} \approx$$
$$\{ x : x \text{ loves } c \text{ in } s \}$$

The Composition Principle Functional Application: The Definite Article

The definite article denotes a relation which is a **function**:

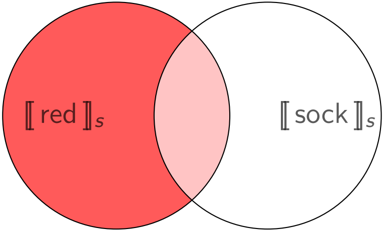
- $[\![\text{the}]\!]_s = \{\langle X, y\rangle : X = \{y\}\}$
- Plugging in would make it (almost) meaningless:

$$[\![\text{the}]\!]_s * [\![\text{moon}]\!]_s = \begin{cases} \text{the empty set} & \text{if } |[\![\text{moon}]\!]_s| \neq 1 \\ [\![\text{moon}]\!]_s & \text{if } |[\![\text{moon}]\!]_s| = 1 \end{cases}$$

- But Functional Application gives the right result:

$$[\![\text{the}]\!]_s([\![\text{moon}]\!]_s) = \begin{cases} \text{undefined} & \text{if } |[\![\text{moon}]\!]_s| \neq 1 \\ ☾ & \text{if } |[\![\text{moon}]\!]_s| = 1 \end{cases}$$

Like nouns, adjectives normally denote sets of individuals, and the extension of the merge of an adjective and a noun is the **intersection** between the two sets:



The pink section is the extension of $[\![\text{red} + \text{sock}]\!]_s$,
$[\![\text{red}]\!]_s \cap [\![\text{sock}]\!]_s$